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Topic List
for the GRE Math Subject Test

This a list of all the topics that I think could conceivably show up on a GRE Math Subject Test. I compiled this list based on my own experiences with the test and what kinds of questions the ETS seems to like to ask (without giving away question specifics, of course!) It's deliberately meant to be somewhat overkill —the more prepared you can be, the better, I'd say!

Note that none of these topics are *guaranteed* to be on the test. (After all, there's only much you can fit in 66 questions anyway!) Even so, the idea is that this should give you a road map for what you might need to brush up on. I've even deliberately included a number of potential "stretch" topics just to try to cover as many bases as possible; these are marked with an asterisk (*).

I hope you find this list useful! Good luck!

—Bill

(P.S.: I also offer personalized tutoring for the GRE Math Subject Test. Contact me via my website, https://www.mathsub.com, if you'd like help reviewing these topics as well as learning some of the tips and tricks I recommend to make them easier!)

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Precalculus

Basic Geometry

- Parallel and perpendicular lines, transversals
- Congruence
- Centers of triangles (circumcenter, incenter, orthocenter, centroid)
- Triangle inequality
- Properties of polygons (angle sum, interior angle measure, etc.)
- Similarity and proportionality
- Angle bisector theorem for triangles
- Pythagorean theorem
- Right triangle trigonometry
- Circles (arcs, chords, inscribed angles, tangents, secants, power of a point, etc.)
- Cyclic quadrilaterals*
- AM-GM inequality*
- Areas and perimeters of triangles (multiple formulas!), quadrilaterals (multiple formulas!), circles, sectors, ellipses
- Volumes and surface areas of cubes, cylinders/prisms, cones/pyramids, spheres, ellipsoids
- Composite figures and shaded regions
- Coordinate geometry (distance, midpoints, etc.)

Basic Algebra

- Basics of functions (domain, range, intervals of increase, end behavior, etc.)
- Algebra of functions
- Inverse functions
- Cyclic functions*
- Functional equations*
- Even and odd functions
- Graphs of equations and transformations
- Solving equations and inequalities (inverse functions, factoring, completing the square, looking at the graph, etc.)
- Lines and linear functions
- Piecewise functions
- Absolute value function
- Floor and ceiling functions
- Max and min functions

Algebraic functions

- Quadratic functions, quadratic formula, discriminant
- Graphs of polynomials
- Binomial theorem and Pascal's triangle
- Factoring and zero-finding techniques (grouping, polynomial division, rational root theorem, Descartes' Rule of Signs, Vieta's formulas, etc.)
- Fundamental theorem of algebra
- Rational functions (asymptotes, holes, etc.)
- Radical functions
- Transforming radicals (rationalizing fractions, radical conjugates, nested radicals)

Transcendental functions

- Exponential functions and exponent laws
- Logarithmic functions and logarithm laws
- Exponential and logarithmic applications and models (growth/decay, Gaussian curves, financial applications, etc.)
- Trigonometric and inverse trigonometric functions
- Circular and harmonic motion
- Laws of sines and cosines
- Trigonometric identities (reciprocal, quotient, Pythagorean, sum/difference, double/half angle, product/sum)
- Combined sinusoids*
- Hyperbolic functions

Analytic Geometry

- Polar coordinates
- Graphs of polar equations (rose curves, limaçons, lemniscates, etc.)
- Plane curves and parametric equations
- Loci in the plane
- Conic sections (circles, ellipses, parabolas, hyperbolas) and their anatomy (foci, eccentricity, etc.)
- Transformations of conics (shifting, scaling, rotating)
- Polar equations of conic sections

Sequences and series

- Sequences and summation notation
- Explicit and recursive formulas
- Factorials
- Arithmetic and geometric sequences and summations

Single-Variable Calculus

Limits

- Concept and definition
- Continuity
- Intermediate Value Theorem and Extreme Value Theorem
- Involving infinity
- L'Hôpital's Rule
- Limits of explicit and recursive sequences

Differentiation

- Tangent lines
- Concept and limit definition
- Differentiability and continuity
- Linearity rules
- Product and quotient rules
- Chain rule
- Higher order derivatives
- Derivatives of elementary functions
- Velocity and acceleration
- Implicit differentiation
- Relative extrema
- Increasing and decreasing functions
- Concavity and points of inflection
- Curve sketching
- Mean Value Theorem
- Optimization
- Related rates
- Logarithmic differentiation
- Parametric and polar derivatives

Integration

- Antiderivatives
- Definite integrals
- Average value and the Mean Value Theorem
- Fundamental Theorem of Calculus
- Leibniz's Rule
- u-substitution

- Area between curves
- Volume: slicing, disks, washers, shells
- Arc length and surface area
- Integration by parts
- Trig substitution
- Reduction formulas
- Partial fractions
- Improper integrals
- Parametric and polar integrals, area, arc length, etc.

Series

- Infinite series
- Geometric series
- Telescoping series
- Integral test
- Comparison test and limit comparison test
- Alternating series and absolute convergence
- Ratio test and root test
- Power series
- Taylor and Maclaurin series
- Common power series
- Remainders of Taylor series and Lagrange's error bound

Multivariable Calculus

Vectors

- Vectors in 2D and 3D, rectangular and polar form
- Dot and cross product
- Lines and planes in space
- Quadric surfaces
- Vector-valued functions
- Vector calculus
- The Frenet frame, curvature, and tortsion*
- Tangent and normal vectors
- Parametric surfaces

Multivariable functions

- Limits and continuity of multivariable functions
- Partial derivatives and differentiability
- Tangent planes
- Multivariable chain rule
- Gradients
- Directional derivatives
- Classifying critical points
- Lagrange multipliers
- Double and triple integrals
- Fubini's Theorem
- Area, volume, and centroids
- Cylindrical and spherical coordinates
- Jacobians

Vector Calculus

- Vector fields
- Line integrals
- Independence of path and conservative vector fields
- Green's Theorem
- Curl and divergence
- Surface integrals and Flux
- Divergence theorem
- Stokes' Theorem*

Differential Equations

First-Order ODE's

- Initial value problems
- Slope fields
- Autonomous ODE's
- Equilibrium solutions
- Separable equations
- Linear equations and integrating factors
- Exact equations
- Inexact equations and integrating factors*
- Solutions by substitutions
- Modeling with first-order ODE's

Higher-Order ODE's

- Reduction of order
- Homogeneous linear equations with constant coefficients
- Undetermined coefficients
- Variation of parameters*
- Cauchy-Euler equations*
- Systems of linear ODE's
- Basic modeling with higher-order ODE's
- Relationships to linear algebra

Stretch topics*

- Laplace transform
- Fourier series
- Partial differential equations

Linear Algebra

Linear equations and matrices

- Systems of linear equations
- Systems of inequalities and linear programming
- Row reduction and echelon forms
- Vector and matrix equations
- Solution sets of linear systems
- Linear independence
- Linear transformations
- Matrix algebra
- Transformation matrices (rotation, dilation, shear, etc.)
- Inverse and invertible matrices
- Triangular and diagonal matrices
- Partitioned and block matrices*
- Matrix factorizations
- Determinants
- Cramer's Rule

Vector spaces

- Definitions
- Subspaces
- Null space and column space
- Bases
- Coordinate systems
- Dimension
- Rank and nullity
- Change of basis
- Inner product, length, and orthogonality
- Cauchy-Schwartz inequality*
- Orthogonal projection
- Other examples of vector spaces (polynomials, functions, etc.) and their linear operators

Eigenvalues etc.

- Eigenvectors and eigenvalues
- Trace
- The characteristic equation
- Cayley-Hamilton Theorem

- Diagonalization
- Minimal polynomials
- Nilpotent and idempotent matrices
- Invariant subspaces and direct sums*
- Jordan normal form*
- Matrix exponentials*
- Gram-Schmidt orthogonalization*
- Least squares solutions*
- Quadratic forms*

Number Theory

Divisibility

- Division algorithm
- GCD and LCM
- Euclidean algorithm
- Diophantine equations
- Fundamental theorem of arithmetic

Modular arithmetic

- Basic properties of congruence
- Base b representations
- Divisibility tricks
- Chinese remainder theorem*
- Fermat's Little Theorem
- Wilson's Theorem

Number-theoretic functions

- Sum and number of divisors
- Euler's phi function
- Euler's theorem

Other topics

- Order of an integer mod m
- Primitive roots
- Quadratic residues*

Abstract Algebra

Groups

- Definitions and properties of groups
- Dihedral groups
- Cyclic groups
- Subgroups
- Permutation groups (including permutation notations)
- Isomorphisms
- Cosets
- Lagrange's theorem
- Cayley's theorem
- Sylow's first theorem*
- Direct products
- Normal subgroups
- Quotient groups
- Homomorphisms
- First isomorphism theorem
- Fundamental theorem of finite abelian groups
- Conjugacy classes
- Automorphism groups*
- Group-like structures: semigroups, monoids*

Rings

- Definitions and properties of rings
- Examples
- Subrings
- Integral domains
- Fields
- Characteristic of a ring
- Ideals
- Quotient rings
- Prime and maximal ideals
- Ring homomorphisms
- Polynomial rings
- Polynomial (ir)reducibility tests*
- Other special types of rings (PID, UFD, Boolean, local, Noetherian, Artinian, etc.)*

Fields

- Field extensions and their relation to vector spaces
- Classifications and structure of finite fields
- Splitting fields*
- Constructible numbers*

Modules*

(Technically mentioned on the syllabus, but this is definitely a stretch topic all unto itself.)

- Definitions and basic examples
- Spanning sets and linear independence
- Free modules
- Invariant basis number condition
- Quotient modules

Discrete Math

Set Theory

- Sets and set operations
- Venn diagrams
- Operations and relations
- Equivalence and order relations
- Functions: injections, surjections, bijections
- Function composition
- Images and preimages
- Cardinality, cardinal numbers, and countability
- Cantor-Schröder-Bernstein theorem

Logic

- Propositional logic and truth tables
- Propositional equivalences
- Predicates and quantifiers
- Proof techniques and proof validity
- Induction

Algorithms

- Basics of pseudocode (input/output, assignment, branching, looping, etc.)
- Growth of functions
- Runtime complexity
- Well-known algorithms
- Recursion*

Combinatorics

- Basics of counting
- Pigeonhole principle
- Binomial and multinomial coefficients
- Permutations and combinations
- Circular permutations
- Complements and the Inclusion-Exclusion Principle
- Generating functions*
- Partitions*

Graph Theory

- Graph terminology
- Graph isomorphism
- Connectivity
- Adjacency matrices
- Euler and Hamilton paths
- Trees
- Tree traversals*
- Spanning trees
- Planar graphs
- Graph coloring*

Probability & Statistics

Probability

- Basic concepts and properties
- Probability and combinatorics
- Probability and geometry
- Conditional probability
- Joint probability
- Independent events
- Bayes's Theorem

Random Variables

- Probability mass/density functions (pmf's/pdf's) and cumulative distribution functions (CDF's)
- Expectation
- Mean, variance, standard deviation
- Moment generating functions (MGF's)*
- Discrete distributions (uniform, Bernoulli, binomial, geometric, Poisson, etc.)
- Continuous distributions (uniform, exponential, normal, t, χ^2)
- Normal approximation of binomial distribution
- Empirical Rule
- Functions of random variables

Statistics

- Mean, median, quartiles, range, percentiles
- z-scores
- Linear regression and correlation
- Linearization of nonlinear models
- Sampling distributions of sample means and proportions
- Point estimation
- Biased and unbiased estimators
- Confidence intervals*
- Hypothesis testing*

Topology

Topological spaces

- Point-set topology and open sets
- Examples (standard, indiscrete, discrete, lower limit, cofinite, etc.)
- Basis for a topology
- Closed sets
- Interior, exterior, boundary
- Limit points, derived set, closure
- Subspace topology
- Product topology
- Quotient topology and "gluing"*
- Geometric examples

Properties of spaces and functions

- Open and closed maps
- Continuous functions
- Homeomorphisms
- Hausdorff property
- Connectedness
- Path connectedness
- Open coverings
- Compactness

Real Analysis

Properties of real numbers

- Supremum/infimum and the completeness property
- Density property
- Archimedean property

Sequences

- Convergence and limits of sequences
- Limit superior and limit inferior
- Bolzano-Weierstrass theorem
- Cauchy sequences

Functions

- Limits of functions
- Continuous functions
- Sequential limits and continuity
- Uniform continuity and continuous extensions
- Other types of continuity (Lipschitz, Hölder, absolute)*
- Bounded variation*
- Differentiability classes
- Riemann and Darboux integrability
- Sequences of functions
- Pointwise and uniform convergence
- Interchange of limits
- Series of functions

Metric spaces

- Metric definitions and examples (Euclidean, taxicab, max, ℓ_p/L_p, etc.)
- Complete metrics
- Topological concepts in metric spaces: compactness, connectedness, continuity
- Heine-Borel Theorem

Lebesgue measure*

(used to be on the test syllabus in ≈1997, likely a stretch topic now)

- Lebesgue measure and measurability
- Basic Lebesgue integration

Complex Analysis

Basics of complex numbers

- Definitions
- Complex plane
- Complex conjugation
- Polar form
- Powers and roots
- Loci and regions in the complex plane

Complex functions

- Functions and mappings
- Linear and fractional-linear mappings
- Inversive geometry*
- Power functions
- Limits and continuity
- Differentiability and analyticity
- Cauchy-Riemann equations
- Harmonic functions
- Elementary functions extended to the complex plane
- Conformal mappings*
- The Riemann mapping theorem*

Complex integration

- Contour integrals
- Cauchy-Goursat theorem
- Maximum Modulus Theorem
- ML Inequality
- Independence of path
- Cauchy's integral formulas
- Laurent series
- Residues and the residue theorem

Numerical Analysis*

(Again, it's mentioned on the syllabus, so may as well be prepared.)

Approximation of functions

- Basic estimation techniques
- Tangent line approximation
- Linear interpolation
- Euler's method
- Taylor polynomial approximations
- Error
- Order of approximations*
- Lagrange interpolating polynomials*

Root finding

- Bisection method
- Fixed points and contraction mapping theorem*
- Newton-Raphson method
- Secant method

Calculus methods

- Finite differences
- Numerical differentiation
- Riemann sums
- Trapezoidal sums
- Simpson's rule